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Advanced Aerodynamics

Homework 2

1. In cylindrical coordinates we introduce the variables

$$
\begin{aligned}
r &= (x^2+y^2)^{\frac{1}{2}}, && (1)\\
\theta &= tan^{-1}(\frac{y}{x}), && (2)\\
z &= z. && (3)
\end{aligned}
$$

Let $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_z$ represent the radial, circumferential and z-axis unit vectors, then the velocity field can be written as

$$\mathbf{V} = u_r\mathbf{e}_r + u_\theta\mathbf{e}_\theta + u_z\mathbf{e}_z. \tag{4}$$

(a) For an inviscid steady flow, the momentum equation is a balance between inertia and pressure forces

$$(\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p \tag{5}$$

Evaluate the radial component of the acceleration $(\mathbf{u}\cdot\nabla)\mathbf{u}$ and write down the radial momentum equation.

(b) If $u_r = 0$, show that (5) reduces to

$$\frac{\partial p}{\partial r} = \rho\frac{u_\theta^2}{r}. \tag{6}$$

Explain this simple result.

(c) For simplicity we assume (i) the flow to be incompressible, i.e., ρ is constant and (ii) $u_z = 0$. Calculate the variation of the pressure for (i) a rigid body rotation, $u_\theta = \Omega r$ and (ii) a free vortex flow, $u_\theta = \Gamma/r$. Compare the result with Bernoulli's equation and explain similarity and difference.

(d) A simple model for a hurricane is to assume a rigid body rotation inside the eye of the hurricane $r < a$ and a free vortex flow outside $r > a$. The two are matched at $r = a$ where the pressure and velocity are assumed to be continuous. Determine Ω and Γ in terms of the pressure at the hurricane center p_0 and at infinity p_∞.

(e) Apply this model to the case of a real hurricane. *Hint: Get information from real data.*

2. Every particle of a mass of liquid is revolving uniformly about a fixed axis, the angular speed varying as the nth power of the distance from the axis. Show that the motion is irrotational only if $n + 2 = 0$.
   If a very small spherical portion of the liquid is suddenly solidified, prove that it will begin to rotate about a diameter with an angular velocity $(n + 2)/2$ of that with which it was revolving about the fixed axis.

3. A hovercraft is designed to float on an air cushion. Assume the 'air cushion' to be contained within a canvas-like skirt that is fastened around the periphery of the hovercraft. Air, supplied by a compressor to the cushion, escapes through the clearance between the end of the skirt and the ground. The skirt has a circular shape of diameter 6m. The mass of the hovercraft is 9000 kg and the average ground clearance is 2.5 cm.The volume of the cushion is sufficiently large so that the velocities in the central portion of the cushion are very low. The air may be treated as incompressible ($\rho = 1.2kg/m^3$), but this assumption should be checked.

   (a) Determine the airflow rate needed to maintain the cushion.

   (b) Find the power given by the compressor to the air cushion.

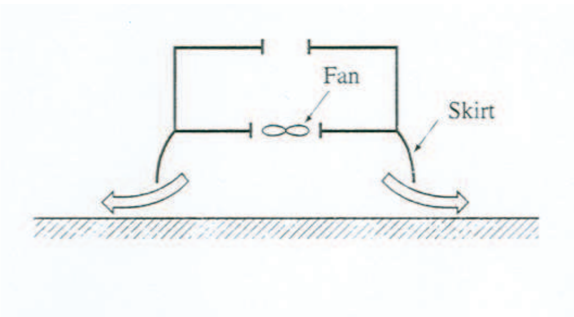

Figure 1: Schematic of a hovercraft